This book is intended for a one-year graduate course on Lie groups and Lie algebras. The book goes beyond the representation theory of compact Lie groups, which is the basis of many texts, and provides a carefully chosen range of material to give the student the bigger picture. The book is organized to allow different paths through the material depending on one's interests. This second edition has substantial new material, including improved discussions of underlying principles, streamlining of some proofs, and many results and topics that were not in the first edition. For compact Lie groups, the book covers the Peter–Weyl theorem, Lie algebra, conjugacy of maximal tori, the Weyl group, roots and weights, Weyl character formula, the fundamental group and more. The book continues with the study of complex analytic groups and general noncompact Lie groups, covering the Bruhat decomposition, Coxeter groups, flag varieties, symmetric spaces, Satake diagrams, embeddings of Lie groups and spin. Other topics that are treated include symmetric function theory, the representation theory of the symmetric group, Frobenius–Schur duality and GL(n) × GL(m) duality with many applications including some in random matrix theory, branching rules, Toeplitz determinants, combinatorics of tableaux, Gelfand pairs, Hecke algebras, the "philosophy of cusp forms" and the cohomology of Grassmannians. An appendix introduces the reader to the use of Sage mathematical software for Lie group computations.
makes an inspiring and simple presentation of the material.
This book has grown out of a set of lecture notes I had prepared for a course on Lie groups in 1966. When I lectured again on the subject in 1972, I revised the notes substantially. It is the revised version that is now appearing in book form. The theory of Lie groups plays a fundamental role in many areas of mathematics. There are a number of books on the subject currently available —most notably those of Chevalley, Jacobson, and Bourbaki—which present various aspects of the theory in great depth. However, I feel there is a need for a single book in English which develops both the algebraic and analytic aspects of the theory and which goes into the representation theory of semi-simple Lie groups and Lie algebras in detail. This book is an attempt to fill this need. It is my hope that this book will introduce the aspiring graduate student as well as the nonspecialist mathematician to the fundamental themes of the subject. I have made no attempt to discuss infinite-dimensional representations. This is a very active field, and a proper treatment of it would require another volume (if not more) of this size. However, the reader who wants to take up this theory will find that this book prepares him reasonably well for that task.
This book starts with the elementary theory of Lie groups of matrices and arrives at the definition, elementary properties, and first applications of cohomological induction, which is a recently discovered algebraic construction of group representations. Along the way it develops the computational techniques that are so important in handling Lie groups. The book is based on a one-semester course given at the State University of New York, Stony Brook in fall, 1986 to an audience having little or no background in Lie groups but interested in seeing connections among algebra, geometry, and Lie theory. These notes develop what is needed beyond a first graduate course in algebra in order to appreciate cohomological induction and to see its first consequences. Along the way one is able to study homological algebra with a significant application in mind; consequently one sees just what results in that subject are fundamental and what results are minor.
Polished lecture notes provide a clean and usefully detailed account of the standard elements of the theory of Lie groups and algebras. Following nineteen pages of preparatory material, Part I (seven brief chapters) treats "Lie groups and their Lie algebras"; Part II (seven chapters) treats "complex semi-simple Lie algebras"; Part III (two chapters) treats "real semi-simple Lie algebras". The page design is intimidatingly dense, the exposition very much in the familiar "definition/lemma/proof/theorem/proof/remark" mode, and there are no exercises or bibliography. (NW) Annotation copyrighted by Book News, Inc., Portland, OR
This is the soft cover reprint of the English translation of Bourbaki's text Groupes et Algèbres de Lie, Chapters 7 to 9. It covers the structure and representation theory of semi-simple Lie algebras and compact Lie groups.
This book is intended for graduate students in Physics, especially Elementary Particle Physics. It gives an introduction to
group theory for physicists with a focus on Lie groups and Lie algebras. An excellent introduction to the theory of Lie groups and Lie algebras. Devoted to the theory of Lie algebras and algebraic groups, this book includes a large amount of commutative algebra and algebraic geometry so as to make it as self-contained as possible. The aim of the book is to assemble in a single volume the algebraic aspects of the theory, so as to present the foundations of the theory in characteristic zero. Detailed proofs are included, and some recent results are discussed in the final chapters. The theory of unitary group representations began with finite groups, and blossomed in the twentieth century both as a natural abstraction of classical harmonic analysis, and as a tool for understanding various physical phenomena. Combining basic theory and new results, this monograph is a fresh and self-contained exposition of group representations and harmonic analysis on solvable Lie groups. Covering a range of topics from stratification methods for linear solvable actions in a finite-dimensional vector space, to complete proofs of essential elements of Mackey theory and a unified development of the main features of the orbit method for solvable Lie groups, the authors provide both well-known and new examples, with a focus on those relevant to contemporary applications. Clear explanations of the basic theory make this an invaluable reference guide for graduate students as well as researchers. This book addresses Lie groups, Lie algebras, and representation theory. The author restricts attention to matrix Lie groups and Lie algebras. This approach keeps the discussion concrete, allows the reader to get to the heart of the subject quickly, and covers all of the most interesting examples. From the reviews: "Sure to become a standard textbook for graduate students in mathematics and physics with little or no prior exposure to Lie theory." --L'Enseignement Mathematique This book reproduces J-P. Serre's 1964 Harvard lectures. The aim is to introduce the reader to the "Lie dictionary": Lie algebras and Lie groups. Special features of the presentation are its emphasis on formal groups (in the Lie group part) and the use of analytic manifolds on p-adic fields. Some knowledge of algebra and calculus is required of the reader, but the text is easily accessible to graduate students, and to mathematicians at large. This book provides quick access to the theory of Lie groups and isometric actions on smooth manifolds, using a concise geometric approach. After a gentle introduction to the subject, some of its recent applications to active research areas are explored, keeping a constant connection with the basic material. The topics discussed include polar actions, singular Riemannian foliations, cohomogeneity one actions, and positively curved manifolds with many symmetries. This book stems from the experience gathered by the authors in several lectures along the years and was designed to be as self-contained as possible. It is intended for advanced undergraduates, graduate students and young researchers in geometry and can be used for a one-semester course or independent study. Lie groups and Lie algebras have become essential to many parts of mathematics and theoretical physics, with Lie algebras a central object.
of interest in their own right. This book provides an elementary introduction to Lie algebras based on a lecture course given to fourth-year undergraduates. The only prerequisite is some linear algebra and an appendix summarizes the main facts that are needed. The treatment is kept as simple as possible with no attempt at full generality. Numerous worked examples and exercises are provided to test understanding, along with more demanding problems, several of which have solutions. Introduction to Lie Algebras covers the core material required for almost all other work in Lie theory and provides a self-study guide suitable for undergraduate students in their final year and graduate students and researchers in mathematics and theoretical physics.

Epstein presents the fundamental concepts of modern differential geometry within the framework of continuum mechanics. Divided into three parts of roughly equal length, the book opens with a motivational chapter to impress upon the reader that differential geometry is indeed the natural language of continuum mechanics or, better still, that the latter is a prime example of the application and materialisation of the former. In the second part, the fundamental notions of differential geometry are presented with rigor using a writing style that is as informal as possible. Differentiable manifolds, tangent bundles, exterior derivatives, Lie derivatives, and Lie groups are illustrated in terms of their mechanical interpretations. The third part includes the theory of fiber bundles, G-structures, and groupoids, which are applicable to bodies with internal structure and to the description of material inhomogeneity. The abstract notions of differential geometry are thus illuminated by practical and intuitively meaningful engineering applications.

From the reviews of the French edition: "This is a rich and useful volume. The material it treats has relevance well beyond the theory of Lie groups and algebras, ranging from the geometry of regular polytopes and paving problems to current work on finite simple groups having a (B,N)-pair structure, or ‘Tits systems’". --G.B. Seligman in MathReviews.

Describing many of the most important aspects of Lie group theory, this book presents the subject in a ‘hands on’ way. Rather than concentrating on theorems and proofs, the book shows the applications of the material to physical sciences and applied mathematics. Many examples of Lie groups and Lie algebras are given throughout the text. The relation between Lie group theory and algorithms for solving ordinary differential equations is presented and shown to be analogous to the relation between Galois groups and algorithms for solving polynomial equations. Other chapters are devoted to differential geometry, relativity, electrodynamics, and the hydrogen atom. Problems are given at the end of each chapter so readers can monitor their understanding of the materials. This is a fascinating introduction to Lie groups for graduate and undergraduate students in physics, mathematics and electrical engineering, as well as researchers in these fields.

This book is intended as an introductory text on the subject of Lie groups and algebras and their role in various fields of mathematics and physics. It is written by and for researchers who are primarily analysts or physicists, not algebraists or geometers. Not that we have eschewed the algebraic and geometric developments. But we wanted to present them in a concrete way and to show how the subject interacted with physics, geometry, and mechanics. These interactions are, of course, manifold; we have discussed many of them here-in particular, Riemannian geometry, elementary particle physics, symmetries of differential equations, completely integrable Hamiltonian systems, and spontaneous symmetry breaking. Much of the material we have treated is standard and widely available; but we have tried to steer a course between the descriptive approach such as found in Gilmore and Wybourne, and the abstract mathematical approach of Helgason or Jacobson. Gilmore and Wybourne address themselves to the physics community whereas Helgason and Jacobson address themselves to the mathematical community. This book is an attempt to synthesize the two points of view and address both audiences simultaneously. We wanted to present the subject in a way which is at once intuitive, geometric, applications oriented, mathematically
rigorous, and accessible to students and researchers without an extensive background in physics, algebra, or geometry. This text introduces upper-level undergraduates to Lie group theory and physical applications. It further illustrates Lie group theory’s role in several fields of physics. 1974 edition. Includes 75 figures and 17 tables, exercises and problems. The book is intended for graduate students of theoretical physics (with a background in quantum mechanics) as well as researchers interested in applications of Lie group theory and Lie algebras in physics. The emphasis is on the inter-relations of representation theories of Lie groups and the corresponding Lie algebras. This book provides an account of part of the theory of Lie algebras most relevant to Lie groups. It discusses the basic theory of Lie algebras, including the classification of complex semisimple Lie algebras, and the Levi, Cartan and Iwasawa decompositions. This volume provides a comprehensive treatment of basic Lie theory, primarily directed toward graduate study. The text is ideal for a full graduate course in Lie groups and Lie algebras. However, the book is also very usable for a variety of other courses: a one-semester course in Lie algebras, or on Haar measure and its applications, for advanced undergraduates; or as the text for one-semester graduate courses in Lie groups and symmetric spaces of non-compact type, or in lattices in Lie groups. The material is complete and detailed enough to be used for self-study; it can also serve as a reference work for professional mathematicians working in other areas. The book's utility for such a varied readership is enhanced by a diagram showing the interdependence of the separate chapters so that individual chapters and the material they depend upon can be selected, while others can be skipped. The book incorporates many of the most significant discoveries and pioneering contributions of the masters of the subject: Borel, Cartan, Chevalley, Iwasawa, Mostow, Siegel, and Weyl, among others. This (post)graduate text gives a broad introduction to Lie groups and algebras with an emphasis on differential geometrical methods. It analyzes the structure of compact Lie groups in terms of the action of the group on itself by conjugation, culminating in the classification of the representations of compact Lie groups and their realization as sections of holomorphic line bundles over flag manifolds. Appendices provide background reviews. A self-contained introduction to the cohomology theory of Lie groups and some of its applications in physics. This classic graduate text focuses on the study of semisimple Lie algebras, developing the necessary theory along the way. The material covered ranges from basic definitions of Lie groups to the classification of finite-dimensional representations of semisimple Lie algebras. Lie theory, in its own right, has become regarded as a classical branch of mathematics. Written in an informal style, this is a contemporary introduction to the subject which emphasizes the main concepts of the proofs and outlines the necessary technical details, allowing the material to be conveyed concisely. Based on a lecture course given by the author at the State University of New York at Stony Brook, the book includes numerous exercises and worked examples and is ideal for graduate courses on Lie groups and Lie algebras. A comprehensive and modern account of the structure and classification of Lie groups and finite-dimensional Lie algebras, by internationally known specialists in the field. This Encyclopaedia volume will be immensely useful to graduate students in differential geometry, algebra and theoretical physics. This collection contains papers conceptually related to the classical ideas of Sophus Lie (i.e., to Lie groups and Lie algebras). Obviously, it is impossible to embrace all such topics in a book of reasonable size. The contents of this one reflect the scientific interests of those authors whose activities, to some extent at least, are associated with the International Sophus Lie Center. We have divided the book into five parts in accordance with the basic topics of the papers (although it can be easily seen that some of them may be attributed to several parts
simultaneously). The first part (quantum mathematics) combines the papers related to the methods generated by the concepts of quantization and quantum group. The second part is devoted to the theory of hypergroups and Lie hypergroups, which is one of the most important generalizations of the classical concept of locally compact group and of Lie group. A natural harmonic analysis arises on hypergroups, while any abstract transformation of Fourier type is generated by some hypergroup (commutative or not). Part III contains papers on the geometry of homogeneous spaces, Lie algebras and Lie superalgebras. Classical problems of the representation theory for Lie groups, as well as for topological groups and semigroups, are discussed in the papers of Part IV. Finally, the last part of the collection relates to applications of the ideas of Sophus Lie to differential equations.

From the reviews: "..., the book must be of great help for a researcher who already has some idea of Lie theory, wants to employ it in his everyday research and/or teaching, and needs a source for customary reference on the subject. From my viewpoint, the volume is perfectly fit to serve as such a source, ... On the whole, it is quite a pleasure, after making yourself comfortable in that favourite office armchair of yours, just to keep the volume gently in your hands and browse it slowly and thoughtfully; and after all, what more on Earth can one expect of any book?" --The New Zealand Mathematical Society Newsletter

A great book ... a necessary item in any mathematical library. --S. S. Chern, University of California A brilliant book: rigorous, tightly organized, and covering a vast amount of good mathematics. --Barrett O'Neill, University of California This is obviously a very valuable and well thought-out book on an important subject. --Andre Weil, Institute for Advanced Study The study of homogeneous spaces provides excellent insights into both differential geometry and Lie groups. In geometry, for instance, general theorems and properties will also hold for homogeneous spaces, and will usually be easier to understand and to prove in this setting. For Lie groups, a significant amount of analysis either begins with or reduces to analysis on homogeneous spaces, frequently on symmetric spaces. For many years and for many mathematicians, Sigurdur Helgason's classic Differential Geometry, Lie Groups, and Symmetric Spaces has been--and continues to be--the standard source for this material. Helgason begins with a concise, self-contained introduction to differential geometry. Next is a careful treatment of the foundations of the theory of Lie groups, presented in a manner that since 1962 has served as a model to a number of subsequent authors. This sets the stage for the introduction and study of symmetric spaces, which form the central part of the book. The text concludes with the classification of symmetric spaces by means of the Killing-Cartan classification of simple Lie algebras over $\mathbb{C}$ and Cartan's classification of simple Lie algebras over $\mathbb{R}$, following a method of Victor Kac. The excellent exposition is supplemented by extensive collections of useful exercises at the end of each chapter. All of the problems have either solutions or substantial hints, found at the back of the book. For this edition, the author has made corrections and added helpful notes and useful references. Sigurdur Helgason was awarded the Steele Prize for Differential Geometry, Lie Groups, and Symmetric Spaces and Groups and Geometric Analysis.

Lie groups has been an increasing area of focus and rich research since the middle of the 20th century. In Lie Groups: An Approach through Invariants and Representations, the author's masterful approach gives the reader a comprehensive treatment of
the classical Lie groups along with an extensive introduction to a wide range of topics associated with Lie groups: symmetric functions, theory of algebraic forms, Lie algebras, tensor algebra and symmetry, semisimple Lie algebras, algebraic groups, group representations, invariants, Hilbert theory, and binary forms with fields ranging from pure algebra to functional analysis. By covering sufficient background material, the book is made accessible to a reader with a relatively modest mathematical background. Historical information, examples, exercises are all woven into the text. This unique exposition is suitable for a broad audience, including advanced undergraduates, graduates, mathematicians in a variety of areas from pure algebra to functional analysis and mathematical physics.

This is the softcover reprint of the 1975 English translation of the first three chapters of Bourbaki’s Groupes et algèbres de Lie. The first chapter describes the theory of Lie algebras, their deviations, representations, and enveloping algebras. Chapter two introduces free Lie algebras in order to discuss the exponential, logarithmic and the Hausdorff series. Chapter three deals with the theory of Lie groups over \( \mathbb{R} \) and \( \mathbb{C} \) ultrametric fields.

As an introduction to fundamental geometric concepts and tools needed for solving problems of a geometric nature using a computer, this book fills the gap between standard geometry books, which are primarily theoretical, and applied books on computer graphics, computer vision, or robotics that do not cover the underlying geometric concepts in detail. Gallier offers an introduction to affine, projective, computational, and Euclidean geometry, basics of differential geometry and Lie groups, and explores many of the practical applications of geometry. Some of these include computer vision, efficient communication, error correcting codes, cryptography, motion interpolation, and robot kinematics. This comprehensive text covers most of the geometric background needed for conducting research in computer graphics, geometric modeling, computer vision, and robotics and as such will be of interest to a wide audience including computer scientists, mathematicians, and engineers.

Lie Groups Beyond an Introduction takes the reader from the end of introductory Lie group theory to the threshold of infinite-dimensional group representations. Merging algebra and analysis throughout, the author uses Lie-theoretic methods to develop a beautiful theory having wide applications in mathematics and physics. A feature of the presentation is that it encourages the reader’s comprehension of Lie group theory to evolve from beginner to expert: initial insights make use of actual matrices, while later insights come from such structural features as properties of root systems, or relationships among subgroups, or patterns among different subgroups.

This textbook treats Lie groups, Lie algebras and their representations in an elementary but fully rigorous fashion requiring minimal prerequisites. In particular, the theory of matrix Lie groups and their Lie algebras is developed using only linear algebra, and more motivation and intuition for proofs is provided than in most classic texts on the subject. In addition to its accessible treatment of the basic theory of Lie groups and Lie algebras, the book is also noteworthy for including: a treatment of the Baker–Campbell–Hausdorff formula and its use in place of the Frobenius theorem to establish deeper results about the relationship between Lie groups and Lie algebras motivation for the machinery of roots, weights and the Weyl group via a concrete
and detailed exposition of the representation theory of $sl(3;\mathbb{C})$ an unconventional definition of semisimplicity that allows for a rapid development of the structure theory of semisimple Lie algebras a self-contained construction of the representations of compact groups, independent of Lie-algebraic arguments The second edition of Lie Groups, Lie Algebras, and Representations contains many substantial improvements and additions, among them: an entirely new part devoted to the structure and representation theory of compact Lie groups; a complete derivation of the main properties of root systems; the construction of finite-dimensional representations of semisimple Lie algebras has been elaborated; a treatment of universal enveloping algebras, including a proof of the Poincaré–Birkhoff–Witt theorem and the existence of Verma modules; complete proofs of the Weyl character formula, the Weyl dimension formula and the Kostant multiplicity formula. Review of the first edition: This is an excellent book. It deserves to, and undoubtedly will, become the standard text for early graduate courses in Lie group theory ... an important addition to the textbook literature ... it is highly recommended. — The Mathematical Gazette

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